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Comments on Quantum-Electrodynamic Corrections to the Electron Charge in Metals*

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Several arguments are presented which cast doubt on the validity of Nordtvedt's conclusion that quantum-electrodynamic corrections to the electron charge produce observable effects which are different *in vacuo* and in a metal.

Nordtvedt[1] has recently concluded on the basis of a random-phase-approximation calculation of the dielectric constant of a metal, taking into account virtual positron-electron pairs, that the electron charge in a metal differs from the free-electron value by an amount of order $10^{-10}e$. If correct, this conclusion would appear to have implications for the exactness of the Josephson frequency-voltage relation.[2] We wish to present several arguments which cast doubt on the validity of Nordtvedt's conclusion.

The physical basis of Nordtvedt's effect is that in a metal, virtual excitation of positron-electron pairs is quenched by the Pauli principle for the occupied electron states below the Fermi energy, while these states are available *in vacuo*. This quenching leads to a decrease of the dielectric constant and hence to an increase of the electron charge in the metal. Nordtvedt's calculation was carried out for an infinite medium. We suggest that when boundary effects are taken into account for a finite medium, the total electron charge, including surface effects, is equal to the electron charge *in vacuo*.

Consider first the familiar example of a dielectric sphere containing a point charge Q at its center. The electric field in the interior of the sphere is $E = Q/\epsilon r^2$. The force due to Q on a test charge inside the sphere can be viewed as the force *in vacuo* due to a renormalized charge $Q' = Q/\epsilon$. The field outside the sphere is Q/r^2. The force on a test charge outside the sphere therefore corresponds to the "true" charge Q of the sphere. The difference $Q - Q'$ appears as a bound polarization charge at the surface of the sphere. If the sphere were infinite in size, physical measurements would not reveal that Q' was not the actual charge. The different situation which occurs when measurements are made outside is directly attributable to the boundary of the sphere and the polarization charge on that boundary. This example suggests that boundary effects must be taken into account in discussing measurable effects of charge renormalization for a finite specimen of a metal.

In a metal it is useful to distinguish two contributions to the dielectric function. The first is the conventional screening which reduces the average electric field to zero in the bulk of the specimen. Clearly, this screening process conserves total charge and is of no direct interest here. The second contribution is the vacuum polarization. Nordtvedt's calculation concerns the modification of this contribution in the presence of the Fermi sea of electrons. If one makes a local dielectric constant approximation for the vacuum polarization effects, the situation is the direct analog of that discussed above. Thus, we would expect the apparent reduction in the renormalized charge within the metal to be compensated by a change in the bound surface charge on the specimen. The addition of an electron to a finite metal sample should increase the over-all charge by exactly e, the electronic charge as measured in vacuum. The effect calculated by Nordtvedt should be exactly cancelled by the surface contribution to the vacuum polarization charge. While nonlocality effects will alter the detailed distribution of the surface charge, we believe that they will not alter this result.

As an illustration, consider two concentric isolated neutral spherical shells, one of lead and the other of aluminum. By Gauss's law, the field exterior to the spheres is zero. If Nordtvedt's arguments were correct, an electric field should appear when an electron is transferred from one shell to the other. The existence of a boundary effect of the type we have attempted to make plausible would reduce that field to zero.

Another argument which casts some doubt on Nordtvedt's conclusion is that atoms, in particular hydrogen, would have a small net charge if his mechanism is applied to such systems. This would follow from the fact that positron-electron pair effects which screen the nuclear charge would be blocked through the Pauli principle by the orbital electron, while the proton would not block the pairs involved in screening the orbital electron charge. Since the density of the 1s electron in the atom is even greater than the mean electron density in metals, on the basis of Nordtvedt's arguments one would naively expect a net charge greater than $10^{-10}e$ to reside on a hydrogen atom. This conclusion is in conflict with the results of several recent experiments[3–6] which place upper limits on the net charge of hydrogen and other atoms in the neighborhood of $10^{-20}e$.

Finally, we would like to emphasize that the question of possible charge renormalization in metals is meaningful only insofar as it relates to observable phenomena and that therefore it must really be discussed in the context of specific experimental procedures. Consider, for example, a black box from which emerge two terminals. Suppose we are told there is a dc potential difference between the two terminals and are asked to determine it with precision. This can be done by connecting to the terminals a circuit containing some device for detecting the flow of electric charge, a reference source of emf such as an electrochemical standard cell, and various metallic conductors. Such a collection of devices might be called a potentiometer. One then adjusts the circuit until the charge-flow detector indicates that no current flows in the measuring circuit. The potential to be measured is then related to the reference emf and determinable parameters of the measuring circuit. The fundamental element in this procedure is the establishment and precision verification of zero charge flow in a closed metallic circuit. In this state the net change in electrochemical potential μ around the circuit is taken to be zero, $\oint \nabla\mu \cdot d\vec{r} = 0$. Implicit in this statement is the assumption that μ is the electrochemical potential associated with a type of entity which is identifiable in all parts of the circuit, so that the electrochemical potential is definable at all points in the circuit. This entity is usually called an "electron." The experiment we have described does nothing more or less than establish the relationship between the electrochemical potential differences across the terminals of our black box and the terminals of the reference source of emf. Supposing this source is a standard cell, we then look at the official National Bureau of Standards calibration report on our standard cell and find its emf stated as x volts. Using this value we then claim to have obtained a result for the potential difference between the terminals of our black box in volts. We have actually done no such thing. The quotation of the standard cell emf in volts is based on the assumption that the electrostatic potential difference V between the terminals of the standard cell is related to the electron electrochemical potential difference $\Delta\mu$ between those terminals by $eV = \Delta\mu$, where e is the charge of an *isolated free electron*. This is an assumption unsupported by direct experiment with any significant precision! We have experimentally compared "electron" electrochemical potential differences, *not* electrostatic potential differences. The conventional quotation of the result in volts rests on an assumed relation between "electron" electrochemical potential and electrostatic potential. It is at this point that the electron charge enters, quite independently of what is in the black box.

Suppose now we are told that inside the black box is a quantum system and a monochromatic source of electromagnetic radiation of frequency ν. The quantum system has the property that two "electrons" entering the black box with electrochemical potential $2\mu_1$ at terminal 1 can make a transit through the box, undergoing an energy-conserving process in which the electron pair emits or absorbs a photon of energy $h\nu$. The two electrons then emerge at terminal 2 with electrochemical potential $2\mu_2$. Then $h\nu = 2|\mu_2 - \mu_1|$ or $h\nu = 2\Delta\mu$. If this process can be detected and monitored experimentally we can combine it with the electrochemical potential comparison experiment described above and relate ν to the electrochemical potential difference across the terminals of a standard cell, for example. If we make the *additional* assumption that this is equal to eV, we have the usual Josephson frequency-voltage relation $\nu = 2eV/h$. The process we have described is identifiable in black boxes containing some type of Josephson junction, various combinations of superconducting and normal wires, microwave frequency sources, etc. The point of our argument here is that the charge e which appears in the Josephson frequency-voltage relation is not operationally identifiable with any specific component in the experimental system, including the Josephson junction or the reference standard cell. The fundamental quantity in such a measurement is the "electron" electrochemical potential, and its dissection into a charge and an electrostatic potential has no operational significance. For other discussions of the electrochemical potential in connection with the Josephson effect see Parker *et al.*[7] and McCumber.[8]

In connection with this last point, we note the following: Consider a normal conductor in which a dc current is flowing so that there is a nonzero electrostatic potential inside the conductor, and consider the process of carrying an "electron" around a closed path which lies partly outside the conductor and partly inside. If we are not to be able to construct a perpetual motion machine, $\oint \nabla\mu \cdot d\vec{r} = 0$ around the path. If it were true that outside the electron has charge e while inside it has $e + \delta e$, and we identify μ with eV outside and $(e + \delta e)V$ inside, then it is easy to imagine a situation where $\oint \nabla V \cdot d\vec{r} \neq 0$.

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Absence of Quantum-Electrodynamic Corrections to the Charge of the Electron as Measured in Josephson Junction Experiments

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The ac Josephson effect enables one to measure the ratio $\Delta\mu/h$, where $\Delta\mu$ is the electrochemical potential difference between two weakly coupled superconductors. Combined with an independent measurement of the electrostatic potential difference V, this measurement determines the ratio e/h. This determination rests on the fact that a voltage difference is defined so that the electrochemical potential difference is $\Delta\mu = eV$, where e is the charge on the free electron. Further, while we argue that it is not directly relevant to the determination of e/h, there are no quantum-electrodynamic corrections to the electrostatic term in the chemical potential difference $\Delta\mu$.

The recent improvement in experimental accuracy[1-3] of the measurements of e/h through the ac Josephson effect lends new importance to efforts to determine the theoretical accuracy of this technique. Recently, for example, Nordtvedt[4] has argued that quantum-electrodynamic corrections will result in an e/h ratio from the Josephson measurements which depends upon the metal used in the junction and differs typically by parts in 10^{-9} from the same ratio for a free electron. We would like to argue here that in fact there are no such quantum-electrodynamic corrections to be expected. In particular, Nordtvedt's results (i) are not directly relevant to the Josephson measurement, (ii) do not in fact change the electrostatic part of the electrochemical potential, and (iii) represent a well-known difference between the $\vec{q} \to 0$ and the $\vec{q} = 0$ limit of the dielectric properties of a many-body system.

The present theories of superconductivity[5] predict that for a superconductor in equilibrium, the time rate of change of the gauge-invariant phase of the order parameter and the electrochemical potential[6] are related by

$$\frac{\partial\varphi}{\partial t} = -\frac{2\mu}{\hbar} . \qquad (1)$$

This implies that if the relative phase between two regions of a superconductor is stationary, the two regions have the same electrochemical potential. Furthermore, if the relative phase changes, then the difference in electrochemical potential is set by (1). Ultimately it is upon the validity of Eq. (1) that the recent solid-state determinations of e/h rest. Briefly,[2] two superconductors are joined by a weak link such as a tunnel junction or thin bridge. Then the rate of change of the relative phase difference across this weak link is synchronized at a multiple $n\omega$ of the frequency ω of an applied microwave field. According to (1) this sets a difference in the electrochemical potential $\Delta\mu$ between the two superconductors which is faithfully propagated by the superconductors to the normal leads. Then a null potentiometer measurement compares this frequency-induced $\Delta\mu$ with the electrochemical potential of a standard cell which is calibrated in absolute volts times the free-electron charge.[7] Thus, the electrochemical potential and not some effective interaction charge of an electron in a metal is what enters the Josephson relation (1) and the measurement. From this it is clear that if quantum electrodynamics is to give corrections to the measurement they must modify Eq. (1).

It might appear that Nordtvedt's prediction of a shift in the charge of an electron in a metal would modify the electrostatic part of the chemical potential difference. Even if this were true, we would argue that it would not affect the determination of the e/h ratio because, as we have discussed, and as has been emphasized in Ref. 2, the Josephson condition establishes a relationship between frequency and chemical potential difference. This re-